Math 10A
Worksheet, Discussion #17; Tuesday, 7/10/2018
Instructor name: Roy Zhao

1 Fundamental Theorem of Calculus II

1. True **FALSE** $\int_a^x f(u)du$ gives you a general form of an antiderivative (including the $+C$).

2. **TRUE** False Let $F(x) = \int_0^x f(u)du$. Then $G(x)$ be another antiderivative of $f(x)$. For all x we have $F(x) = G(x) - G(0)$.

3. If $\int_1^x f(u)du = \frac{1}{x} + a$, find f, a.

Solution: Taking the derivative, the left side gives us $f(x)$ and the right side gives us $-x^{-2}$ so $f(x) = -x^{-2}$. Then we have that $\int_1^x f(u)du = 1/x - 1/1$ so $a = 1$.

4. Find $\frac{d}{dx} \int_1^x \ln t dt$.

Solution: $\ln x$ by FTC.

5. Find $\frac{d}{dx} \int_x^3 e^{se^s} dx$.

Solution: $-e^{xe^x}$.

6. Find $\frac{d}{dt} \int_2^{t^2} \sqrt{1 - x^3} dx$.

Solution: $\sqrt{1 - (t^2)^3} \cdot \frac{d}{dt} t^2 = 2t\sqrt{1 - t^6}$.

7. Find $\frac{d}{dx} \int_{2x}^{x^3} \frac{t}{2t+1} dt$.

Solution: $\frac{x^3}{2x^3 + 1} \cdot 3x^2 - \frac{2x}{2(2x) + 1} \cdot 2 = \frac{3x^5}{2x^3 + 1} - \frac{4x}{4x + 1}$.

Substitution Rule

8. Find $\int \frac{\ln x}{x} dx$.

Solution: Let $u = \ln x$, then $du = \frac{dx}{x}$, so we have that

$$\int \frac{\ln x}{x} dx = \int u du = \frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C.$$

9. Find $\int x\sqrt{1-x} dx$.

Solution: Let $u = 1 - x$ and so $du = -dx$ and $x = 1 - u$ so this is

$$\int x\sqrt{1-x} = \int (1-u)\sqrt{u}(-du) = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C = \frac{2(1-x)^{5/2}}{5} - \frac{2(1-x)^{3/2}}{3} + C.$$

10. Find $\int_0^{\sqrt{\pi}} x\cos(x^2) dx$.

Solution: Let $u = x^2$ and so $du = 2xdx$ and when $x = 0$, then $u = 0$ and when $x = \sqrt{\pi}$, then $u = \pi$ so we have

$$\int_0^{\sqrt{\pi}} x\cos(x^2) dx = \int_0^{\pi} \frac{\cos(u) du}{2} = \frac{\sin u}{2}|_0^{\pi} = 0.$$

11. Find $\int \sin(x)\sec^2(x) dx$.

Solution: We rewrite $\sec^2(x) = \frac{1}{\cos^2(x)}$. Let $u = \cos(x)$ so that $du = -\sin x dx$ and hence

$$\int \sin(x)\sec^2(x) dx = \int -u^{-2} du = \frac{1}{u} + C = \frac{1}{\cos(x)} + C = \sec(x) + C.$$

12. Find $\int 2xe^{e^{x^2}} e^{x^2} dx$.

Solution: We first try $u = x^2$ so $du = 2xdx$ and hence

$$\int 2xe^{e^{x^2}}e^{x^2}dx = \int e^{e^u}e^u du.$$

Now let $v = e^u$ so $dv = e^u du$ and hence

$$= \int e^v dv = e^v + C = e^{e^u} + C = e^{e^{x^2}} + C.$$

13. Find $\int xe^{x^2}dx$.

Solution: Let $u = x^2$, then $du = 2xdx$ and hence $xdx = \frac{du}{2}$. Therefore

$$\int xe^{x^2}dx = \int \frac{e^u du}{2} = \frac{e^u}{2} + C = \frac{e^{x^2}}{2} + C.$$

14. Find $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$.

Solution: Let $u = \sqrt{x}$ so $du = \dfrac{dx}{2\sqrt{x}}$. Then

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}} = \int 2\sin u du = -2\cos u + C = -2\cos\sqrt{x} + C.$$